THE UNIVERSITY OF MELBOURNE

Journal Club

Complexity and Procedural Choice

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Introduction

Paper

James Banovetz & Ryan Oprea, 2023. "**Complexity and Procedural Choice**," American Economic Journal: Microeconomics, American Economic Association, vol. 15(2), pages 384-413, May.

1. Research Question
2. Two-arm Bandit Problem
3. Experimental Design
4. Automata
5. Pattern Recognizing
6. Results

Research Question

Does complexity make people adjust their procedure of decision-making?

(from more complex, optimal procedures to simpler but suboptimal procedures

Authors advocates an algorithmic-level theory that uses **finite state automata (FSA)** to explains individual decision-making procedure. They conducted experiments with **two-arm bandit problem** to support this theory.

- Assumption (0) people indeed use FSA procedure to solve the problem
- Assumption (1) people try to avoid complex procedures (with higher state-complexity i.e. more states)

The authors manipulated the cognitive costs imposed on subjects and tested if subject behaved differently when under low costs vs. high costs.

Two-Arm Bandit Problem I: Setup

Two-arm bandit problem

1st Option

$payoff = x$, with certainty

$0 < x < 1$

2nd Option

$payoff = y$, with certainty

But, $y = \begin{cases} 0, & \text{prob} = {}^{1}/_{3} \\ x, & \text{prob} = {}^{1}/_{3} \\ 1, & \text{prob} = {}^{1}/_{3} \end{cases}$

Repeated, each period with probability $(1-\delta)$ to end

- The first arm chosen by the DM would become 1st option(with fixed payment x), no matter which arm it was.
- The other arm, therefore, would become 2nd option (with fixed payment y)

Two-Arm Bandit Problem II: Solution

With assumption of basic rationality, if DM decides to explore the 2nd option, her following actions follow:

1. If it yields 1, she should stick to 2nd option
2. If it yields 0, she should return to 1st option

$$\mathbb{E}[\text{explorative rule}] = \frac{1}{3}\Big(\underbrace{\Big(0 + \delta \cdot \frac{x}{1-\delta}\Big)}_{\substack{\text{Found out 2nd option (at cost)} \\ \text{and switch back to 1st option}}} + \frac{x}{1-\delta} + \underbrace{\frac{1}{1-\delta}}_{\substack{\text{Found out 2nd option} \\ \text{and keep exploiting it}}}\Big)$$

$$\mathbb{E}[\text{non}-\text{explorative rule}] = \frac{x}{1-\delta}$$

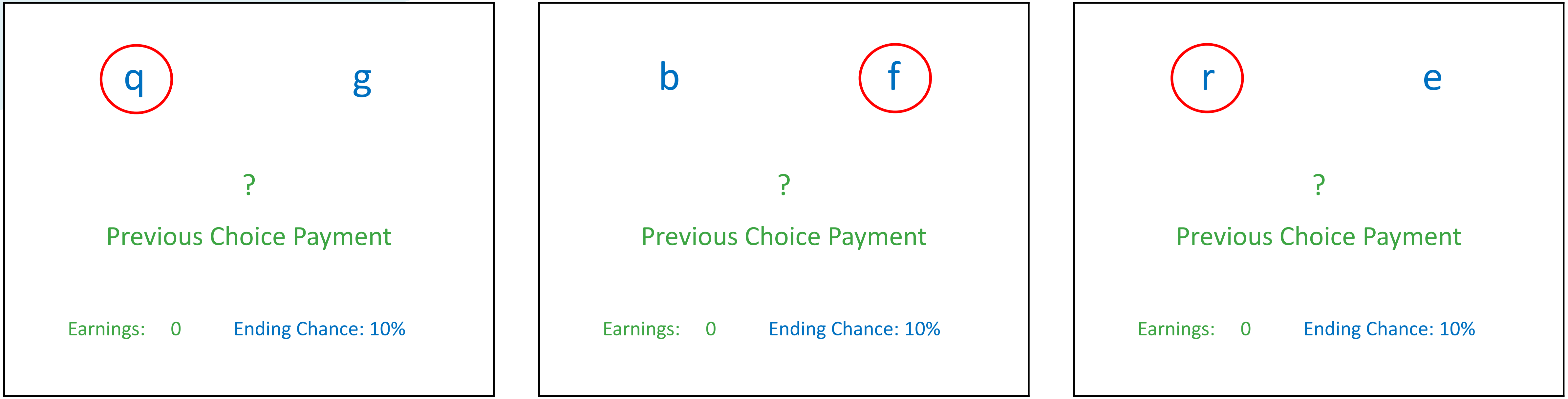
When is exploring the optimal choice, in contrast to exploiting 1st option without exploring 2nd option?

$$\frac{1}{3}\Big(\Big(0 + \delta \cdot \frac{x}{1-\delta}\Big) + \frac{x}{1-\delta} + \frac{1}{1-\delta}\Big) > \frac{x}{1-\delta}$$

$$\Rightarrow x < \frac{1}{2-\delta}$$

Set up $\begin{cases} x = 0.65 \\ \delta = 0.9 \end{cases} \Rightarrow$ optimal to explore

Experiment Design: Task I



- Options represented by letters
- Choose by typing
- Once selected an arm, (the latest) payment is shown
- Cumulative earnings update
- Ending chance

Randomization of:

- Letter representation
- Position/orientation

⇒ Alphabetically earlier letter option vs. Alphabetically later letter option

Experiment Design: Task II

Task 1 Task 2 Task 3 … Task 20

Period 1 Period 2 … Period n

Each period with a prob. of 10% to be the last (in this task)

Block

Played in blocks of 5 period

- In each task, participants will always play multiple of 5 times of periods
- If a period is selected to be the ending one, participants only know after finishing this whole block
- Any period after the ending period does not count toward final payoff

Experiment Design: Implementation

Deployment: **Qualtrics**

Recruiting platform: **Prolific**

Subjects: 180, all from the US

Time: Mostly 20-30 mins

Incentive: $2.5 show-up fee + bonus based on performance

- Average points across all tasks
- Hurdle at 700 pts
- Every point > 700 -> $0.03
- >95% participants earned extra
- On average, $5.05 and $7.14 for two main treatments

Instruction → **Comprehension** → **Main Session**

Experiment Design: Treatments

1. **State Tracking (ST)**

2. **Non State Tracking (NST)**

q (65)

g (?)

65

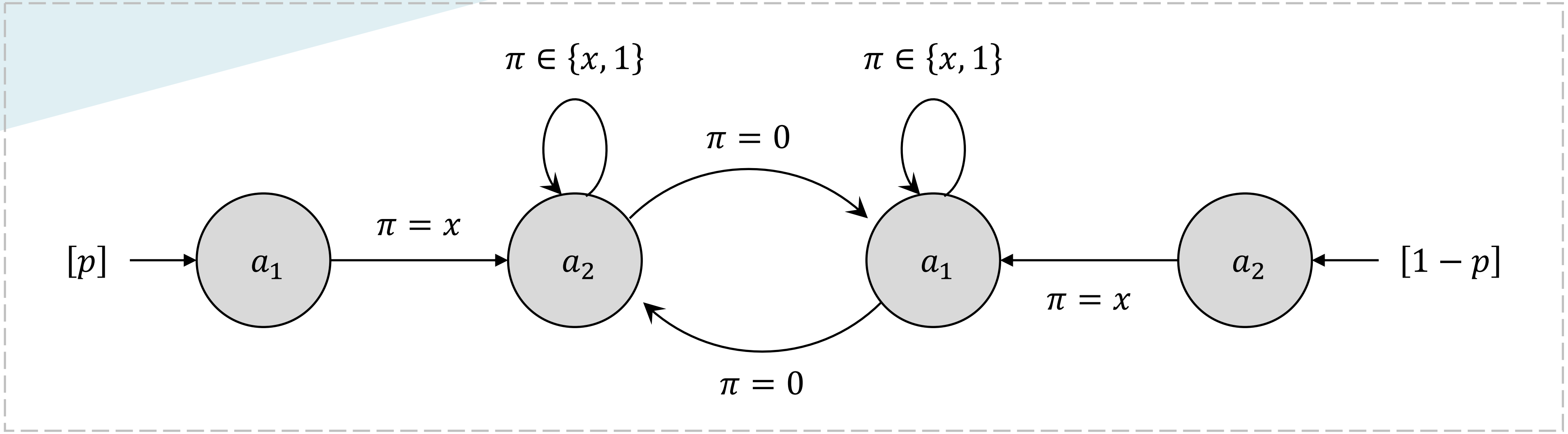
Previous Choice Payment

Earnings: 65 Ending Chance: 10%

3. **State Tracking + Distraction (ST+D)**
   - Random letter flashed “between” (before?) each period
   - Type in all 5 letters at the end of each block correctly to earn another 300 pts
   - Additional monetary reward hurdles at 1000 pts

4. **NST-ST**
   - NST instruction + 10 NST & ST instruction + 10 ST

Automata I: Optimal, Randomized

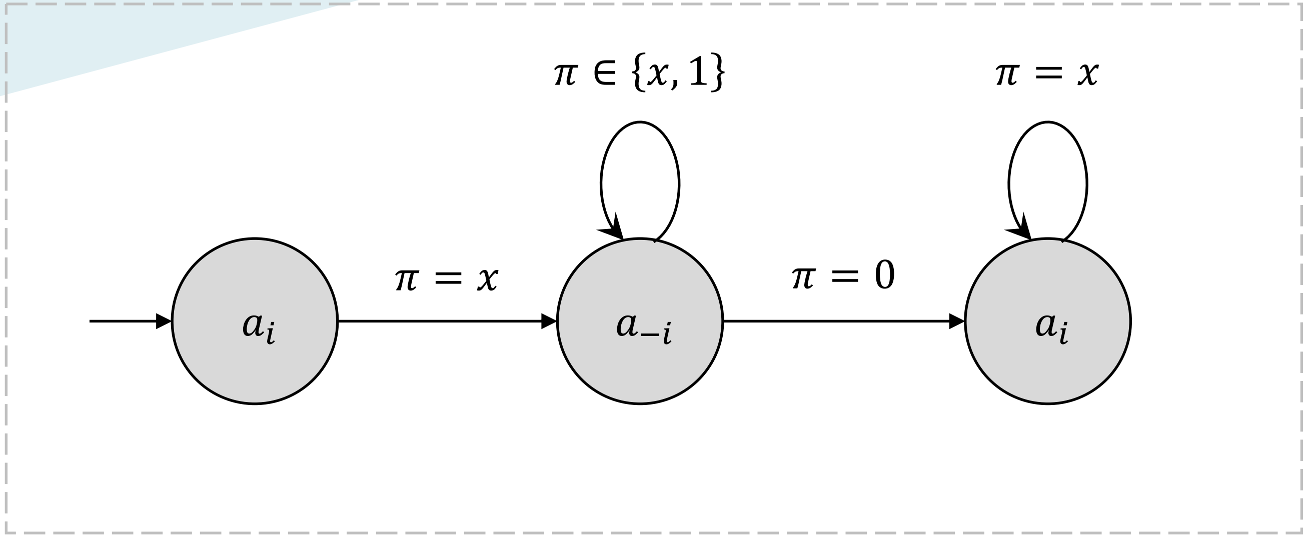


(one form of) the optimal solution/algorithm: **4-state automata**

Why do we need 4 states?

- Tracking past actions – Where she started and where she ended up at
- Tracking past events – How she ended up where she's at
    - at 1st option: just started and has not explored 2nd option vs. explored and switched back
    - at 2nd option: just explored and has not yet switched vs. explored and decided to stay

Automata II: Optimal, Hard-Coded

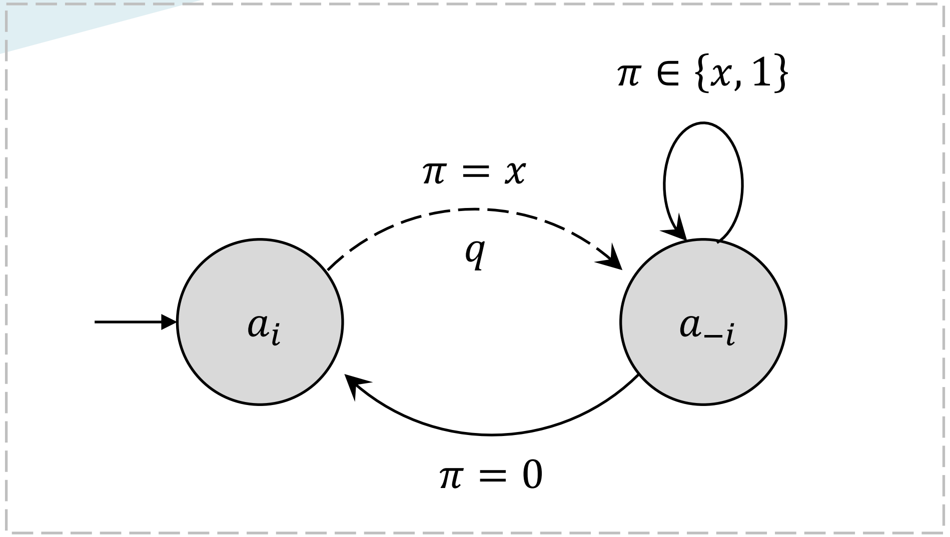


(one form of) the optimal solution/algorithm: **3-state automata**

Reduced complexity by not tracking initial action to avoid randomization.

- Do not care to track initial selection of arm
- Hard-coded
- "Costless" – without losing optimality

Automata III: Suboptimal, Mixed

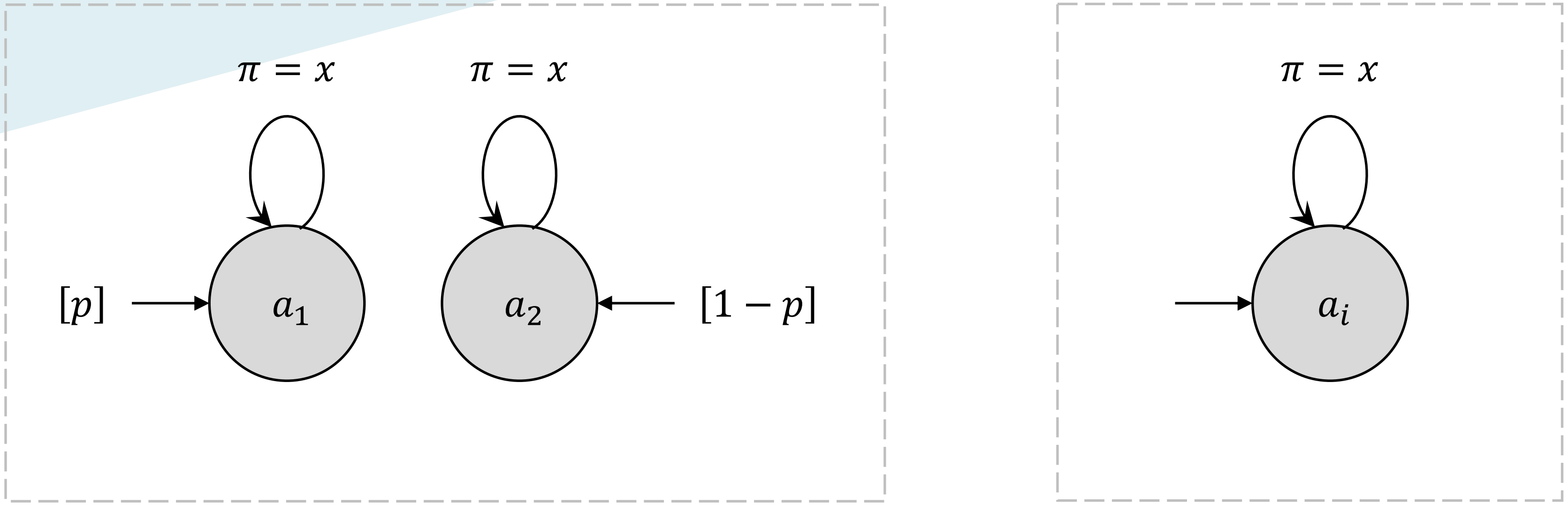


A suboptimal solution/algorithm: **2-state automata**

Further reduced complexity by ignoring past information.

- Not remembering payoff from 2nd option
- Offload memory
- “Costly” – losing optimality

Automata IV: Suboptimal, Non-Explorative

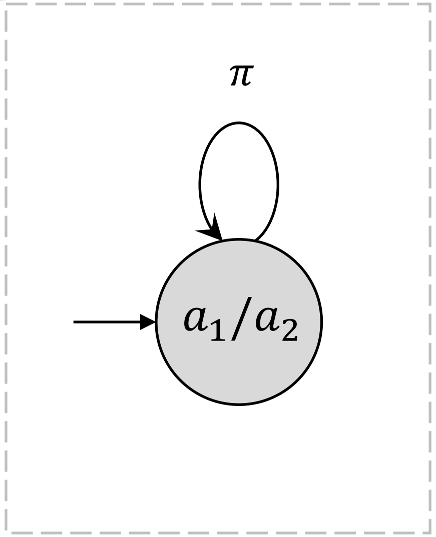


A suboptimal solution/algorithm: **2-state automata**

Reduced complexity by ignoring one option and giving up potential action.

- Not explore at all
- Avoided uncertainty
- "Costly" – losing optimality

Automata V: Suboptimal, Non-Tracking



A suboptimal solution/algorithm: **1-state automata**

Radically reduced complexity by playing randomly.

- Give up using brain…thus, easiest to play
- Lowest expected payoff

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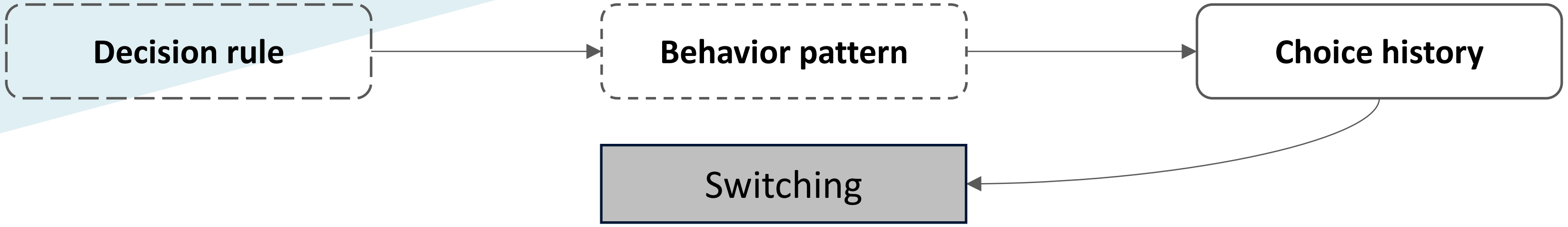
Behavioral Fingerprints I: Key Features



Preceding action

Which arm DM just selected:
- *1A*: the option she chose initially
- *2A*: the option she did not choose initially

Past events

Whether DM has explored and what she found out:
- *I*: she has not explored yet
- *0, x, 1*: the payoff of 2nd option (explored)

	DM switches…
1A-I	from 1st option when she has not explored i.e. explore right after initial selection
1A-0	from 1st option after knowing 2nd option yields 0
2A-0	from 2nd option after knowing 2nd option yields 0
2A-1	from 2nd option after knowing 2nd option yields 1
*H**	Consistently choosing the same arm as 1st option

Behavioral Fingerprints II: Theoretically

Basic rationality principle:

- Switch away from 2nd option if it pays 0
- Never switch away from 2nd option if it pays 1

Optimal procedures:

- Basic rationality principle holds
- Always switch away from 1st option initially
- Never switch away from 1st option if 2nd option pays 0

Mixed procedure:

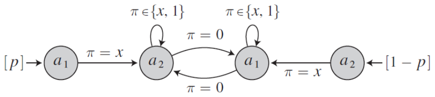
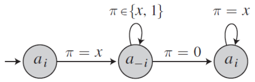
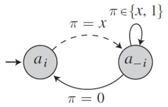
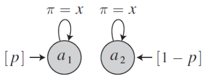
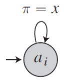
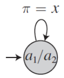
- Basic rationality principle holds
- Switch away from 1st option but with constant probability q (both initially and when 2nd option pays 0) since not remembering past events

Non-explorative procedures:

- Never switch away from 1st option

Non-tracking/random procedures:

- Switch with ~50% chance

Rule	Automata	1A-I	1A-0	2A-1	2A-0	H
Optimal, Randomized *Four-state*		1	0	0	1	No
Optimal, Hard-coded *Three-state*		1	0	0	1	Yes
Mixed *Two-state*		q	q	0	1	–
Non-explore, Randomized *Two-state*		0	–	–	–	No
Non-explore, Hard-coded *One-state*		0	–	–	–	Yes
Non-tracking *One-state*		≈ 0.5	≈ 0.5	≈ 0.5	≈ 0.5	–

Behavioral Fingerprints III: Empirically

Switches at a “high rate” if > 0.75, “low rate” if <0.25, and with “interior likelihood” if in between.

Rule	1A-I	1A-0	2A-1	2A-0	H
Optimal, Randomized *4-state*	high rate	low rate	high rate	low rate	False
Optimal, Hard-coded *3-state*	high rate	low rate	high rate	low rate	True
Mixed *2-state*	diff. at low rate		high rate	low rate	-
Non-explorative, Randomized *2-state*	low rate	-	-	-	False
Non-explorative, Hard-coded *1-state*	low rate	-	-	-	True
Non-tracking *1-state*	interior likelihood	interior likelihood	interior likelihood	interior likelihood	-

Hypotheses

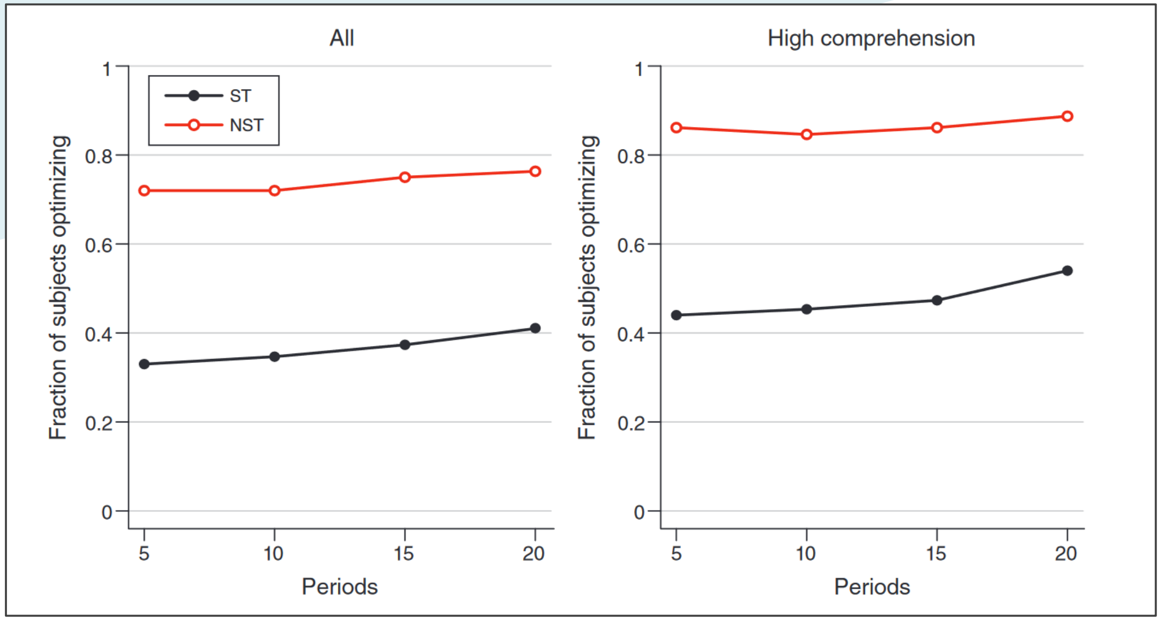
Fundamental: offloading complexity leads to more adoption of more complex procedures

- People don’t like complexity and have a natural tendency to avoid/economize on complexity costs whenever possible.
- When having to track states, which is a costly action, DM avoids complexity at a cost by deviating to lower-state, simpler rule.
- When freed from tracking states, DM can allocate more mental resources to use more complex, optimal rule.

Hypothesis 1: Subjects are **more likely to use optimal procedures in the NST** treatment than in the ST treatment.

Hypothesis 2: Subjects are **more likely to use lower-state versions of Optimal and Non-exploration procedures in the ST** treatment than in the NST treatment.

Results I: Suboptimality



Comprehension check:

- Answer optimal choice to an instance of two-arm bandit arm task
- Repeated trials
- If incorrect, correct answer and explanation pops up

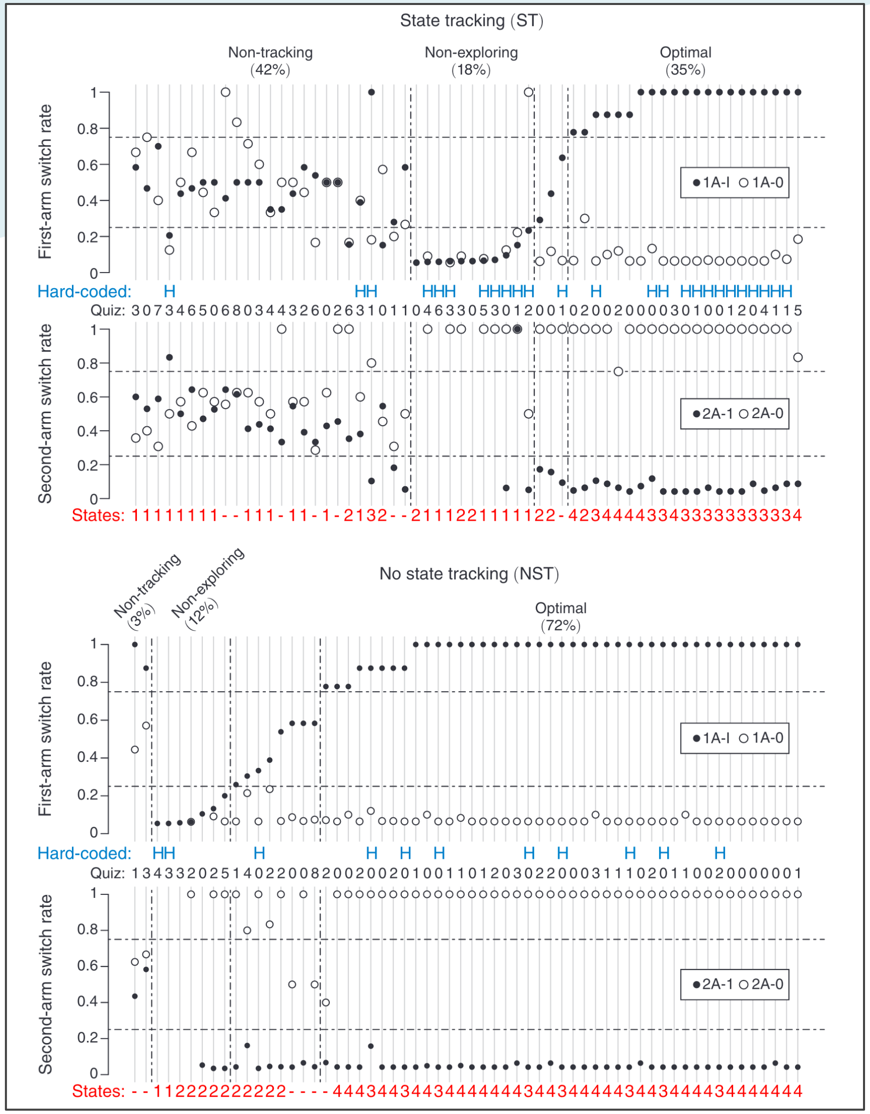
"High-comprehension":

- Those who passed with no more than one incorrect answer

Finding 1: Less than half of subjects in ST made optimal decisions.

Finding 2: In contrast to ST, most subjects behave optimally in NST.

Results II: Individual Behavior Patterns



Finding 3 – Non-tracking: 42% in ST vs. 3% in NST

- subjects switched from all arms

Finding 4 – Non-explorative: 18% in ST vs. 12% in NST

- original version: high-comprehension only, 13% in ST vs. "virtually disappeared" in NST

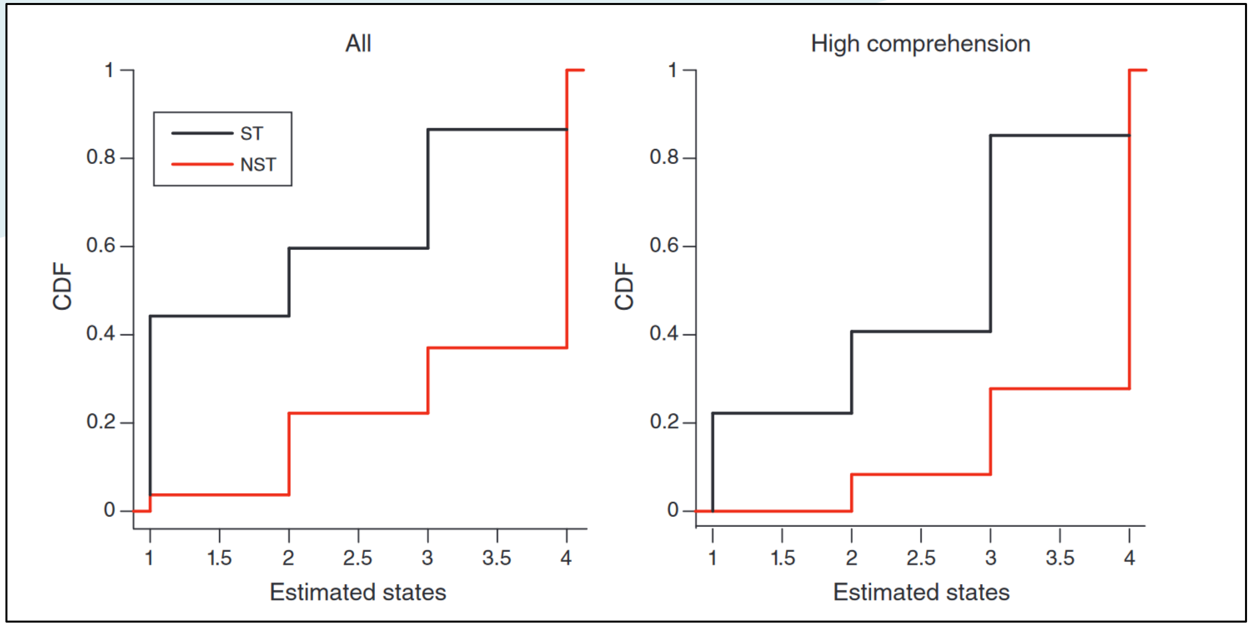
Finding 5 - Optimal: 35% in ST vs. 72% in NST

- cost was removed
- original version: "more than doubled"
- if high-comprehension: 50% in ST vs. 85% in NST

Finding 6 – Hard-coded: 50% in ST vs. 21% in NST

- looked at high-comprehension only
- Corresponds to **Hypothesis 2**

Results III: Classification



Finding 7: Subjects used significantly lower-state procedures when state complexity costs were high (ST) than when they were low (NST).

- Corresponds to **Hypothesis 2**
- K-S test: both cases p < 0.001

Finding 8: Subjects were more than four times as likely to use simplified suboptimal procedures in ST than in NST.

- ~50% in ST vs. <10% in NST
- Proportion test: both cases p = 0.002

Finding 9: Subjects optimized in a systematically different way when exposed to state complexity costs. In ST, 75% of optimizing subjects used simpler 3-state rules, while in NST, 79% used more complex 4-state procedures.

Discussion

1. Strong assumption on algorithmic level theory – people used FSA as the procedure of decision-making in solving two-armed bandit problem
   - Could there be competing explanation that also corresponds to the results of the study?
   - Do people even know/realize that they were solving the problem this way?

2. Psychological/biological explanation of state complexity – how exactly having more states makes one procedure more mentally demanding?
   - Author's explanation: (1) efforts to avoid mistaking contingencies and (2) efforts to encode and recall rules
   - Can we measure or hypothesize this on a more fundamental, say neurological, level?

3. Presentation of the results – switching between using all data and part of the data (high-comprehension subjects only) several times without proper justifications
   - Sticking to using all data would not undermine the results, but should this kind of behaviors be criticized?
   - Although figures themselves show clearly, why not do more regression?

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